Teaching Conceptual Knowledge About Arithmetic Operations Using Chartworld Software: An Evaluation

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Abstract Two fully randomized treatment/control experimental studies tested the effectiveness of Chartworld software on enhancing the mathematical knowledge of third grade children. In each study, 196 children were randomly assigned to receive a total of four hours of instruction (either Chartworld or traditional textbook). All children were given a written test and a structured interview in a pre- post-test design. The first study focused on knowledge of multiplication and division. On both response measures (written and interview), ANCOVA results indicated significant group differences in favor of Chartworld in the magnitude of posttest performance while controlling for initial pretested ability ($p < .05$). The second controlled study covered divisibility and prime and composite numbers. ANCOVA was used to examine posttest performance while controlling for pretested ability. The results demonstrated greater gains in conceptual knowledge for the Chartworld group than for the control group on both of the response measures ($p < .001$). Although the raw effect sizes revealed substantial growth for both groups with respect to conceptual knowledge gained during the course of the study, the Chartworld gains were significantly greater than those made by the control group. Specifically, Chartworld gains were 44% greater on the interview and 67% greater on the written test. These experiments provide corroborating evidence that Chartworld provides an effective method of instructional delivery that yields value-added enhancement to students' conceptual knowledge preferable to what can be gained from traditional textbook instruction. In addition to these quantitative results, we also describe examples of visual patterns which children find particularly interesting. Many of these patterns are mathematically rich, as well as important to the mathematics curriculum. We describe approaches in which teachers can use these features to develop instruction that appeals to children both visually and intellectually. Finally, we present a case study which describes how a successful example of Chartworld instruction was improved through feedback from a mathematician and from classroom teachers. We provide an analysis of specific mathematical concepts; along with directions for further enhancing Chartworld instruction.

Theoretical Perspectives

Chartworld is a flexible computer program that allows children to create a wide variety of patterns. There is a mathematical reason for each pattern, and one of the strengths of the program is that children can explore the patterns at any stage of understanding, and can learn mathematics as they explore. With Chartworld, children can easily create models showing: multiples, the commutative property of multiplication, perfect squares, division as the inverse of multiplication, division with remainder, factors, prime numbers, divisibility tests, and the Sieve of Eratosthenes.

Chartworld developed within a constructivist approach to learning, emphasizing conceptual reasoning based on number sense, in addition to the more traditional focus on rote memorization. Working within this constructivist approach, Seymour Papert (1980) developed Logo, a highly popular computer language during the 1980s. Logo proved to be simple enough for kindergartners to draw shapes and yet powerful enough for college students to write sophisticated programs to solve problems in differential geometry (Abelson and diSessa, 1981). Despite all of the potential benefits, Logo has not gained widespread acceptance in the public school classrooms.

In an effort to build upon Logo's considerable strengths and to correct its weaknesses, Andy diSessa developed Boxer, a programming language where text editing, graphics, and database capability are all integrated with programming.

Chartworld is written in Boxer, which has proved to be flexible enough for students to write programs on a wide range of topics (diSessa 2000; diSessa and Abelson, 1986). Crucially, it is very easy to modify any program in response to requests from teachers and students.

This research incorporated a fully randomized controlled experimental design. Theoretical and empirical evidence converge to support the conclusion that poor performance working with numbers may be largely traced to separation between conceptual understandings of properties about numbers and performing operations with numbers, such as performing arithmetic. Conceptual knowledge can be defined as the awareness of what mathematical symbols mean, and the ability to represent relations among numbers in multiple ways.

Recent reports by the National Academy Press, (e. g. Bransford, Brown, & Cocking, 2000) review the major cognitive science perspectives on information-processing models of mathematics skills. Consistent with these reports, it is necessarily important for children to coordinate conceptual knowledge in order to understand the underlying meanings of arithmetical operations with numbers. Thus, conceptual knowledge reveals the logical structure of any math domain. The ability to understand the meaningfulness of numbers, operations, and applications, is often cited as one of the defining characteristics of the emerging construct in the mathematical cognition literature referred to as “number sense” (e. g. Berch, 2005). Conceptual understandings about whole numbers are important for children to be able to apply their understandings toward solving procedural and word problem tasks. (Hecht 2006; 2002). Consequently, it is necessary to develop quantitative tests that measure children’s knowledge about their understandings of numbers, as well as qualitative methods to describe how children use conceptual understandings in the process of operating with numbers.

Overview of Chartworld Instruction

Chartworld software can be used to teach arithmetic and number theory (Ploger & Della Vedova, 1999; Ploger & Rooney, 2005). Unlike other computer-based methods, Chartworld allows children to create, modify, and extend colorful models which can be explored to understand mathematics in a progressively deeper way. Children use Chartworld to create a variety of visually interesting patterns which express fundamental mathematical ideas. Children can begin exploring using Chartworld before they completely understand these ideas; teachers can teach the mathematical concepts when children are ready.

Chartworld is set up to color all multiples of a number when it is clicked with the mouse. Clicking on the number 2 will color all the multiples of 2. Figure 1 shows the result in a 6-column chart.

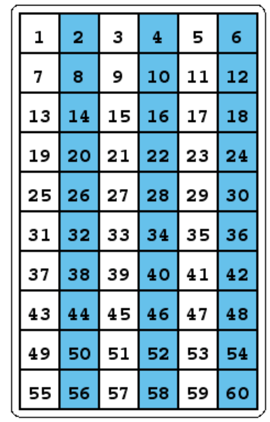

Figure 1: Clicking 2 in a 6-column chart.

In addition to coloring the multiples of 2, children also notice that the visual result is 3 blue “stripes” in a 6-column chart, illustrating the multiplication fact, 2 x 3 = 6. "Stripes" (connected vertical lines of color) constitute one interesting pattern.

Figure 2 shows the result of clicking on 3 after having clicked on 2, illustrating the commutative property of multiplication: 2 x 3 = 6 and 3 x 2 = 6. Children learn that if 2 makes 3 stripes, then 3 must make 2 stripes.

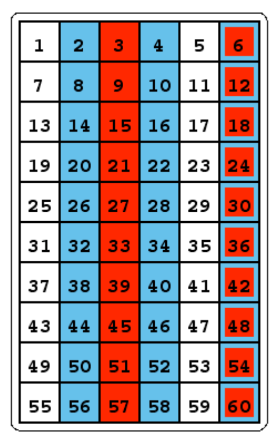

Figure 2: Blue and red stripes in a pattern.

Once these essential concepts of multiplication are understood, children can learn division using the same models. Figure 1 could also be used to show that 6 ÷ 2 = 3. Moreover, children can then connect these multiplication and division facts, realizing that stripes imply an entire fact family.

Children can use these same models to discover the factors of a number, offering a smooth transition to a discussion of prime numbers. To motivate the discovery of prime numbers, children are asked to color every number except 1. Starting at the beginning, children will click on 2 and 3. As they go to click on 4, they see that it is already in color, as is 6. So they must click on 5 and 7. The result of these operations is shown in Figure 3.

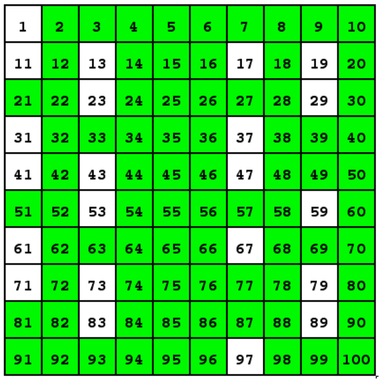

Figure 3: The Sieve of Eratosthenes

The Sieve of Eratosthenes, which is shown in Figure 3, is a classic mathematical concept, used to generate the prime numbers. [Although this challenging topic is ordinarily difficult for children to understand, with Chartworld, children understand the concepts.]

The teacher helps children discover that each of the remaining uncolored numbers in the chart can only be colored by clicking on that number itself (except of course by clicking on 1, which would not work for this exercise, because clicking on 1 would color every number in the chart, including the primes). Together with the numbers 2, 3, 5, and 7, the children have discovered all of the prime numbers less than 100. Students are intrigued by this process.

The simple Sieve shown in figure 3 has many advantages. Children can create it in less than a minute, using the same version of Chartworld that they have used since the lessons began. On the other hand, there are some drawbacks. 2, 3, 5, and 7 have solid color, even though they are all prime, and 1 has no color, even though it is neither prime nor composite.

To address these issues, a multi-color Sieve was created. Figure 4 shows that all the numbers in color are composite, and each color codes for its smallest prime factor. For example, 4 is a composite number with a smallest prime factor (SPF) of 2. It has a solid yellow color, as do all other composite even numbers. (2 is given a yellow border, showing that it is prime, and that it is used to filter out all the composite multiples of 2.)

Figure 4: A Multi-color Sieve of Eratosthenes.

The teacher directs the children to examine the factors of each number. The children learn that the smallest factor (other than 1) of any number must be prime. This is of great value in determining whether a number is prime: it is only necessary to perform divisibility tests using prime numbers. Chartworld gives students the freedom to explore a wide range of mathematical topics on their own, leading them to a mastery of mathematical topics in the curriculum.

Quantitative Results

Two fully randomized controlled studies tested the effectiveness of Chartworld on enhancing the mathematical knowledge of third grade children. For both studies, Chartworld and control groups received teacher-led instruction on topics covered in traditional textbooks, with normal variations that would be expected from teacher-to-teacher in a regular school setting. The control group received instruction directly from the textbook; the Chartworld group used Chartworld examples for the same topics.

The first study focused on conceptual and procedural knowledge of multiplication and division. Children were randomly assigned to one of two conditions: Chartworld instructional or traditional textbook (control). Each group received a total of four hours of instruction. Following instruction, all children were given a written posttest, as well as a structured interview. For the structured interview, the Chartworld group increased 22% on conceptual knowledge from initial to post-test time periods; the control group increased 11%. For the traditional written test, the Chartworld group increased 24% on conceptual knowledge, while the control group showed only a 14% gain. On both outcome measures, ANCOVA's indicated significant differences between groups favoring Chartworld students with respect to mean levels of posttest performance, while controlling for initial pretested performance ($p < .05$).

The second controlled study was conducted approximately three weeks after the State mandated high-stakes testing (i.e., FCAT) was completed by the District. [The second study taught 4th grade mathematics, and the third grade children responded well to this challenge.] The second study focused on conceptual and procedural knowledge of prime and composite numbers (as part of instruction to prepare children to operate with fraction quantities). The third graders received a total of four hours of instruction on prime and composite numbers. As before, children were again randomly assigned to one of two conditions: Chartworld instructional or traditional textbook (control). [In this randomized process, some children were assigned to a different group; some were assigned to the same group. The children actually saw the flip of the coin, and they accepted the rules of the game.] The control group and the Chartworld group received instruction for the same amount of sessions and the same total time. ANCOVA's were employed to examine posttest performance while controlling for pretested ability. The results demonstrated greater gains for the Chartworld group than for the control children on both structured interview and traditional written test measures ($p < .001$). Although the raw effect sizes revealed substantial growth for both groups with respect to conceptual knowledge, the Chartworld gains were 44% greater on the interview and 67% greater on the written test than were the gains of the control children. Thus, Experiment 2 provided corroborating evidence that Chartworld provides an effective method of instructional delivery that enhances student learning.

Qualitative Results

In addition to these quantitative results, we also describe examples of visual patterns which children find particularly interesting. Many of these patterns are mathematically rich, as well as important to the mathematics curriculum. We describe approaches in which teachers can use these features to develop instruction that appeals to children both visually and intellectually. We present a case study which describes how a successful example of Chartworld instruction was improved through feedback from a mathematician and classroom teachers. PrimeRow is an implementation of the Sieve of Eratosthenes in Chartworld, one which encourages children to explore prime and composite numbers. In the original version, the children were asked to guess the rules of the game, where the program showed the coloring scheme before there was enough information to understand that scheme. In the revised version, the coloring was altered to clarify the procedure, making it easier for teachers to explain the ideas. We provide an analysis of specific mathematical concepts; along with directions for further enhancing Chartworld instruction.

PrimeRow: A Case Study

In order to show the relation between the actions of Chartworld and the particular concept being taught, we will explicitly show the relation between the Chartworld event and the concept definition. In discussing the design choices for the behavior that Chartworld displays, we will describe the original version, its success in the classroom, the comments of a mathematical consultant, and the revised version.

The original version was developed in response to requests from teachers and students. An example is shown in Figure 5:

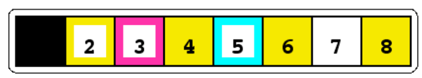

Figure 5: The original version of PrimeRow

In this version, both 3 and 5 have already been given a border. There is a very clear reason: the multiples of 3 will be used to filter out the composites. However, at this stage, there is no way a student would know this. As it turns out, children could deal successfully with this puzzle. However, the program is not nearly as clear as it could be.

In the revised version, the number 2 is not given a yellow border until 4 appears. In proceeding step by step up to 8, as shown in Figure 6:

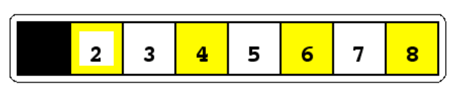

Figure 6: The revised version of PrimeRow.

So far, the pattern has been: 2 has a yellow border; all other even multiples are solid yellow. No odd number has any color. It is certainly reasonable to expect that 9, an odd number, will get no color.

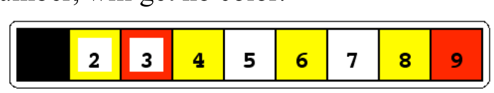

Figure 7: Red color appears.

When 9 appears, some teachers and students have said, “Now that’s interesting!” It is not what most expect. This stage shows that the pattern is not “all odds are white.”

So far, 2 has a yellow border and all other even numbers are solid yellow. 3 has a red border and all other odd multiples of 3 (namely, 9, 15, and 21) are solid red. It is reasonable to expect, following this pattern, that 25 will be white. In the revised version, 25 is solid blue, and 5 has been given a blue border.

The revised version of PrimeRow is based upon the model: Prime numbers filter out the composite numbers. The remaining numbers are prime. When a prime number first appears, it is clear; when a composite number first appears, it has solid color, colored according to its SPF. As it turns out, clicking on 2, 3, and 5 eliminates all the composite numbers, and what remains are the primes.

From Patterns to Mastery

It is clear from our experience in the classroom that children enjoy Chartworld. However, their initial spontaneous explorations are not connected to the central ideas of the elementary school curriculum. For example, most third and fourth graders like to click on 1 repeatedly with different colors.

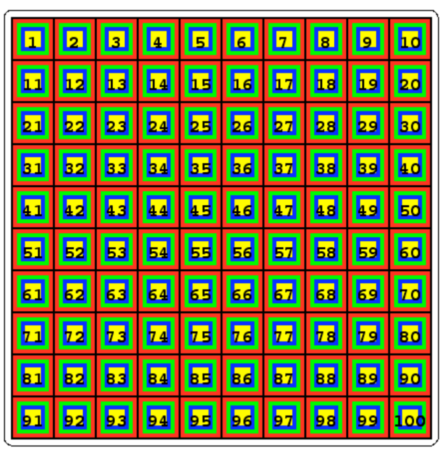

Figure 8: The multiples of 1.

It is not clear how much benefit results from this – and we clearly cannot recommend this activity to conscientious elementary school teachers preparing their students for challenging mathematical topics.

Although it is important to give children some time to appreciate visually rich pictures, we also need to direct that activity toward some mathematically rich experience that will connect to the curriculum. We are looking for a balance. We need to allow exploration without losing direction; we need to exert our instructional authority while preserving an important level of freedom for students.

One especially useful example is the multiples of 2, 3, 4, and 6 in a 12-column chart. Most of the children we have worked with are intrigued by this pattern of multi-colored stripes.

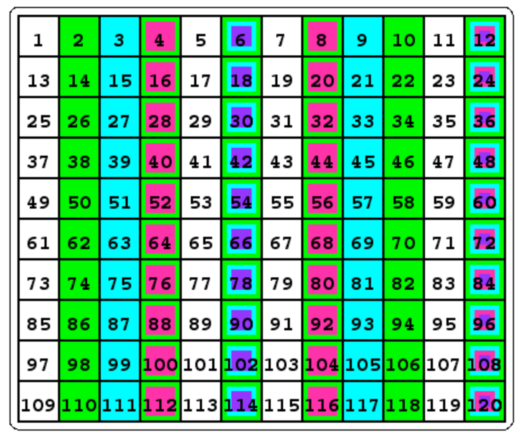

Figure 9: Multiples in a 12-column chart.

When they changed the number of columns to 13, all the stripes turned to diagonals.

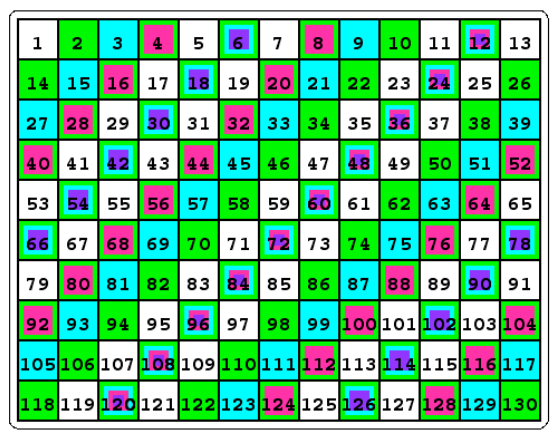

Figure 10: The same multiples in a 13-column chart.

Figures 9 and 10 illustrate an important concept: consecutive numbers have no common factor (other than 1). This is shown in Chartworld every time the number of columns is either increased or decreased by 1, stripes change to another pattern.

It is important to emphasize: children do not automatically discover any of these mathematical ideas by simple exploration. Instruction is needed, but it is not **just** direct instruction. Often, the most effective instruction builds upon the child's spontaneous explorations.

Acknowledgments

Teachers are central to the success of any work in the schools. No matter how good the technology is, the approach must be helpful to the teachers. Our project has been very fortunate in this respect.

We thank Mr. Michael Rooney, who has worked with Chartworld for over a decade. His excellent results with Chartworld in the classroom are an inspiration to us all. We thank Principal, Mr. Charles McCanna for his consistent dedication to his teachers, and for applying his extensive technical skills to help us achieve success. We thank Principals Ms. Peggy Roberts and Ms. Carol Lesser who have been most gracious in with their support. We thank Dr. Kathleen O'Rourke and Mr. Steven Peskin who have helped us coordinate with the schools. We thank Ms. Margaret Nelson and Ms. Sylvia Ramgadoo for their skill and enthusiasm in helping their students learn Chartworld. And, we thank Ms. Mary Beth Mazza and Ms. Deborah Howell, who coordinated the day-to-day work in the schools. Although we will never know how they managed to do all we asked of them, we are most grateful that they did!

Chartworld Software

Chartworld materials were developed by Don Ploger, Associate Professor of Education at Florida Atlantic University. The Chartworld toolset, from which materials were constructed, was created by Andrea A. diSessa in the Boxer programming environment: www.PyxiSystems.com.

Researchers and Educators interested in using Chartworld: please contact Don Ploger: ploger@fau.edu

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